QP – 174

V Semester B.A./B.Sc. Examination, March/April 2022
(CBCS 2016-17 and Onwards) (F+R)
MATHEMATICS
Mathematics (Paper – VI)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions. (5×2=10)

1. a) Show that the shortest distance between two points in a plane is a straight line.

b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} \left[y^2 - (y')^2 - 2y \sin x \right] dx$.

c) Write Euler's equation when the function f is independent of x and y.

d) Evaluate $\int_C [x\,dy - y\,dx]$, where C is the curve $y = x^2$ from (0, 0) to (1, 1).

e) Evaluate $\int_0^a \int_0^b (x^2 + y^2)\, dx\,dy$.

f) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz^2\, dx\,dy\,dz$.

g) State Stoke's theorem.

h) Show that the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab using Green's theorem.

P.T.O.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

b) Find the geodesic on a surface of right circular cylinder.

OR

3. a) Find the extremal of the functional $I = \int_{x_1}^{x_2} \left[12xy + (y')^2\right] dx$.

b) Solve the variational problem $\int_1^2 \left[x^2 (y')^2 + 2y\,(x + y)\right] dx = 0$ with the conditions y (1) = 0 and y (2) = 0.

4. a) Find the extremal of the functional $I = \int_0^{\pi} \left[(y')^2 - y^2\right] dx$ with y (0) = 0 and y (π) = 1 and subject to the constraint $J = \int_0^{\pi} y\, dx = 1$.

b) Show that the general solution of Euler's equation for the integral $I = \int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is $y = ae^{bx}$.

OR

5. a) Find the equation of the plane curve on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.

b) Find the extremal of the functional $I = \int_{x_1}^{x_2} \left[y^2 + (y')^2 + 2ye^x\right] dx$.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Evaluate $\int_C (x + y + z)\, ds$, where 'C' is the line joining the points (1, 2, 3) and (4, 5, 6) whose equations are x = 3t + 1, y = 3t + 2, z = 3t + 3.

b) Evaluate $\oint_C [(x+2y)\,dx+(4-2x)\,dy]$ along the curve $C: \frac{x^2}{9}+\frac{y^2}{4}=1$ in anticlockwise direction.

OR

7. a) Evaluate $\iint_A (4x^2-y^2)\,dxdy$, where 'A' is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

b) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by using double integration.

8. a) Evaluate $\int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c}(x^2+y^2+z^2)\,dxdydz$.

b) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dxdy$ by changing to polar coordinates.

OR

9. a) Evaluate $\iiint_R xyz\,dxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into cylindrical polar coordinates.

b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) State and prove Green's theorem.

b) Evaluate using divergence theorem $\iint_S \vec{F}\cdot\hat{n}\,dS$, where $\vec{F}=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S be the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

OR

11. a) Verify Green's theorem in the plane for $\oint_C \left[xy + y^2\right] dx + x^2 dy$, where C is the closed curve bounded by y = x and $y = x^2$.

b) Evaluate $\iint_S \vec{F} \cdot \hat{n} dS$ using divergence theorem where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the closed surface bounded by planes z = 0, z = 1 and the cylinder $x^2 + y^2 = 4$.

12. a) Evaluate by Stoke's theorem $\oint_C [yzdx + zxdy + xydz]$, where C is the curve $x^2 + y^2 = 1, z = y^2$.

b) Evaluate by using divergence theorem for $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ over the rectangular parallelepiped $0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3$.

OR

13. a) Evaluate by Stoke's theorem $\oint_C [\sin z dx - \cos x dy + \sin y dz]$, where C is the boundary of rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$.

b) Evaluate using Green's theorem in the plane for

$\int_C \left[3x^2 - 8y^2\right] dx + \left[4y - 6xy\right] dy$, where C is boundary of the region enclosed by x = 0, y = 0 and x + y = 1.
